Collective-Adaptive Lévy Flight for Underwater Multi-Robot Exploration

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***Abstract*—This paper presents the use of Lévy flight, a bio-inspired algorithm, to efficiently and effectively locate targets in underwater search scenarios. We demonstrate how a novel adaptation strategy, building on the Firefly optimization algorithm, substantially improves Lévy flight performance. The adaptation strategy represents a swarm intelligence approach, the distribution patterns governing robot motion are optimized in accordance with the distribution of targets in the environment, as detected by and communicated between the robots themselves. Simulation experiments contrasting the performance of the present Lévy flight and two other search strategies in both sparse and clustered distributions of targets are conducted. We identify Lévy flight as exhibiting the best performance, and this is improved with our adaptation strategy, particularly when targets are clustered. Finally, Lévy flight's superior performance over the alternative strategies examined here is empirically confirmed through deployment on real-world underwater swarm robotic platforms.**

I. INTRODUCTION

Unmanned underwater exploration is instrumental in investigating biological species, monitoring pollution, implementing disaster warning systems, and search-rescue missions. An autonomous robot equipped with sensing peripherals is deployed in the underwater environment to locate targets interest, i.e., sea mines, black boxes from downed aircraft or ships, or hazardous chemicals. In a very large environment locating targets through use of a single AUV (Autonomous Underwater Vehicle) is inefficient, and a swarm approach comprising multiple AUVs operating and communicating in parallel can succeed more quickly. A swarm of robots essentially encompass a wireless mobile sensor network, performing distributed sensing in a dynamic environment.

Animals and insects perform foraging activities when searching for food sources in nature. With no knowledge of the environment and a limited sensory range, biological creatures are left with little alternative to a random walk when foraging. With limited energy supplies, the process of evolution and natural selection has lead to animals that optimize their random walk strategies.

There are parallels with underwater exploration: the environment is typically unknown, sensory range is limited, exact localization (e.g. GPS) is often unavailable, and it is difficult to implement reliable long-range global communication channels. At best, AUVs are initialised with a random-walk pattern that can only be optimised when sufficient information concerning the environment is acquired.

Fig. 1. Cocoro 'Lily' AUV platform

Previous work has investigated bio-inspired random walk implemented on robots performing search in a 2-dimensional environment [6]. An artificial repulsive potential field was applied to improve robot dispersion following deployment and increase their separation in the environment. This work, however, assumed that targets were always sparely distributed. For many real-world search applications this is not the case: sea predators forage for prey which are clustered as a group, such as schools of fish. Underwater mines are clustered in fields, not uniformly distributed across oceans. As social creatures, human victims in disaster areas are also typically found in groups. The previous work was restricted to two dimensions, and did not consider cooperation and communication amongst robots. In this work, we demonstrate how a bio-inspired searching mechanism can be supplemented with a strategy allowing the algorithm to adapt to the distribution of targets in the environment, as detected and communicated between robots. Robots following our search strategy move in a particular direction for a set period of time, but the durations of these movements are influenced by encountering targets, or fellow robots who themselves have. The adaptation strategy operates at the collective level, exploiting local communication to share adaptive parameters and turning competition or indifference into beneficial cooperation. This improvement scales with additional robots, as opportunities for communication are increased. It is important to emphasise that the present work is conducted in 3D underwater (both simulated and real-world) environments, furthering the challenge. In these environments in particular a communication-based cooperative strategy is beneficial, as global coordination and localization are problematic.

This work in identifying optimum random search algorithms was borne from the ANGELS [7] AND CoCoRo [8] European Union (EU) project. These projects are motivated by scenarios wherein multiple AUVs cooperate in searching for phenomenon of interest forms some constituent in a larger goal of underwater chemical leakage monitoring, locating metal objects, or docking for reconfigurability [9,10].

In this paper, a 3D underwater swarm simulation platform developed within CoCoRo is used for investigating and comparing the feasibility and the efficiency of several random search algorithms. Simulation holds several advantages for this endeavour, simplifying the means and accuracy of experimental observation, facilitating many repeats of a stochastic experiment, and providing a platform for rapid prototyping and assessment of search strategies. Simulation-based evaluation is followed by deployment of several search strategies in a real-world robotic setting. The experimental underwater swarm robotic platform from the CoCoRo project, named 'Lily' (figure 1), is used in the experimental aquarium for performing simple target location.

The rest of this paper is structured as follows. The Secs. II and III describe theoretical approaches underlying the bio-inspired Lévy flight algorithm and optimization strategy. Secs. IV and V are devoted to implementation and experiments. Finally Sec. VI concludes this work.

II. Lévy Flight Random Search

Lévy flight is a well known biological random search. A Lévy flight random walk pattern uses a Lévy probability distribution that has an infinite second moment, which is advantageous when targets are sparsely and randomly distributed [1,3,4,5].

The Lévy probability distribution has the following form [3,4,12]:

$$P_{\alpha,\gamma}(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\gamma|q|^{\alpha}} cos(ql)dq \tag{1}$$

The distribution is symmetrical around $l = 0$, γ represents the scaling factor and α determines the shape of the distribution. α takes a value between 0 and 2, and determines the shape of the distribution's tail: larger values of α provide shorter tail regions. An α value of 2 shapes the Lévy distribution into a Gaussian distribution. By fixing $\gamma = 1$, for large values of l, (1) can be approximated by [3,4]:

$$P_{\alpha}(l) \approx l^{-(\alpha+1)} \tag{2}$$

Viswanathan et al. [4] derived several analytical solutions to optimize the Lévy flight's parameters. They note that given a priori knowledge about the distribution of target sites, and if sensing is limited, an optimal strategy for a forager is to set $\alpha = 1$.

III. Optimization Algorithm

We review here several well known nature-inspired evolutionary optimization strategies.

In Ant Colony Optimization (ACO) [14], ants employ stigmergy by distributing a chemical pheromone in the environment for establishing shortest path between their nest and the food source. However, implementing a similar mechanism in an underwater context is problematic; chemical marking will be diffused underwater. An alternative such as local communication is easier implement underwater than using the environment for sharing the information among swarm members.

In Particle Swarm Optimization (PSO) [15], a population of particles is maintained which undertake search and/or exploration of a space. Particles (represented as an agent) maintain positions and velocities within a multi-dimensional search space, and update their motion based on the quality of their position in the environment, and the state of the swarm. PSO employs global communication to find the best location occupied by any member during the search process. This communication requirement renders PSO unsuitable for underwater exploration, where global communication and localization is problematic.

There are also two well known algorithms inspired by tropical firefly flashing behaviors. Yang developed an optimization algorithm based on the attractiveness of the flashing fireflies in 2010 [16]. The metaheuristic algorithm was then used for finding optimal solutions in a multimodal optimization problem. A second firefly-inspired algorithm provides synchronization of oscillators has found wide-spread application in communication network applications [17,18].

A firefly's flashing light is generated by a bioluminescence process, and attracts both prey and mates. A firefly's apparent attractiveness is proportional to the intensity of light it generates, thus for any two flashing fireflies, the dimmer firefly will move towards the brighter one. Fireflies have a limited range for this form of communication, as light intensity decreases with distance from the source, and the phenomenon such as fog absorb the light. As such, fireflies are known to perform random walks around the environment when outside of their neighbours communication range.

The Firefly Optimization (FO) algorithm, which mimics the attracting behavior of fireflies, is as follows [16]:

- Step 1: Initialize population with random positions
- Step 2: Define light absorption coefficient μ
- Step 3: Light intensity I_i at x_i is determined by $f(x_i)$
- Step 4: Do for each firefly in swarm
  - Find other fireflies in communication distance
  - if the light intensity of other firefly is higher, move towards its position by using the movement equation

- Update the light intensity

- Step 5: Repeat steps 3-4 until termination condition is reached

The movement of firefly i is attracted to the other with higher light intensity firefly j by using the equation:

$$x_i(n\hat{+}1) = x_i\hat{(n)} + \beta_0 e^{-\mu r_{ij}^2}(\hat{x_j} - \hat{x_i}) + b(rand - 0.5) \quad (3)$$

where:
$\hat{x_i}$ = the position of the firefly i
$\hat{x_j}$ = the position of the firefly j
β_0 = attractiveness of the firefly j
μ = light absorption coefficient
r_{ij} = distance between firefly i and j
b = weight coefficient [0:1]

The second term of the movement equation represents attraction, and the third term is randomization with b representing the random gain parameter. In this original firefly optimization algorithm, uniform distribution is applied to generate random movement, in case the firefly is outside communication range with others, thus results zero for the second term.

We have generated a novel swarm algorithm for exploring an environment that incorporates social interaction, but without the need for global communication or stigmergy. Our algorithm constitutes individual agents performing a Lévy flight exploration, endowed with a short term memory representing their individual past experiences. This memory is defined as attractiveness and is used for local communication between agents when in communication range. Memory and communication are used to modulate the properties of the exploration pattern of other AUVs. In this manner it constitutes a "swarm level" cognitive layer that reflects the constitution of the swarm and the configuration of the environment. The algorithm is based on a modification of the original FO algorithm [16].

The first step of the modification is to choose the attractiveness parameter. An AUV's attractiveness is defined as the period since it last found a target; it increases every time a target is located and decays with time. The attractiveness variable influences an AUV's velocity as follows:

$$v_{n+1} = w_i v_n - c\beta \quad (4)$$

where:
w_i = inertia coefficient
v_n = current velocity of the robot
β = attractiveness of the firefly
c = weight coefficient

The second step of the modification entails adopting a Lévy flight random walk to control motion. The random motion element of the FO is simply replaced by the random number from the Lévy distribution generator. The modified FO algorithm is as follows:

- Step 1: Initialize population with random positions and constant velocities
- Step 2: Evaluate - compute attractiveness variables
  - if the robot locates a target, increase the attractiveness value
  - if the robot does not see any target, apply linear decay to the attractiveness value
- Step 3: Do if the robot agent is in communication range with other nearby robot
  - Step 3.1: compare the attractiveness variable with neighboring robots
  - Step 3.2: move towards the robot that has higher attractiveness
  - Step 3.3: update attractiveness and velocity
- Step 4: Repeat steps 2-3 until termination condition is reached

In combination with the movement equation in the next section, the proposed optimization algorithm can scale the pattern of the random walk relative to the distribution of the found targets. After locating a target an AUV's average forward velocity is decreased, thereby increasing the likelihood that it locates other targets clustered nearby.

The proposed algorithm also influences other robots in the swarm, negotiating and to recalculating the attractiveness, that become an adaptation parameter for scaling the random walk, thus improving the performance in collective level.

Hence, at the collective level, this combined optimization algorithm and bio-inspired random walk becomes more adaptive to distribution of targets in space. It is also important to note that the algorithm performs best if and AUV's communication range (or 'attracting range') is further than its sensory range (for recognizing the availability of a target), as is the case on the CoCoRo 'Lily' AUV.

IV. Implementation Framework

One main goal of this work is to provide a generic implementation framework for the Lévy flight random walk in underwater robotic platforms, and demonstrate this both in simulation and in real robots.

A. *Lévy Distribution Generator*

A numerical approach for generating random numbers based on Lévy probability distribution is required to generate the length of the walk/swim for each robot. Such an algorithm was introduced in [12]. This algorithm requires two independent random variables a and b which have Gaussian distribution. Furthermore, a nonlinear transformation function is introduced:

$$m = \frac{a}{|b|^{\frac{1}{\alpha}}} \quad (5)$$

within the nonlinear transformation, the sum of variables with an appropriate normalization

$$z_n = \frac{1}{n^{\frac{1}{\alpha}}} \sum_{k=1}^{n} m_k \quad (6)$$

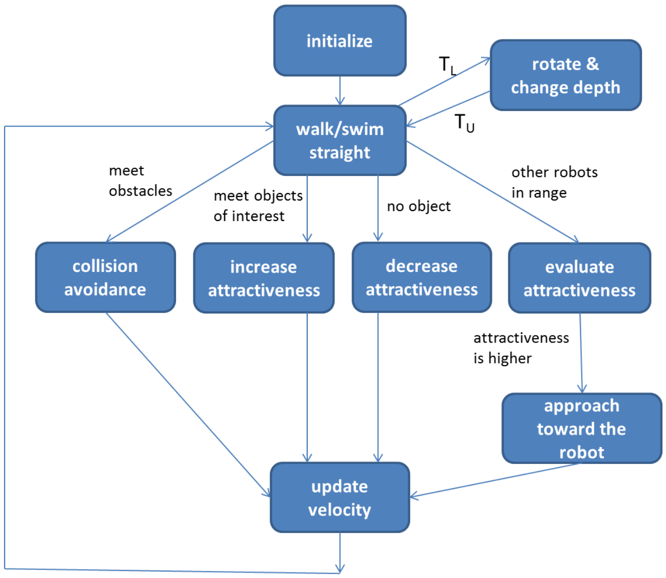
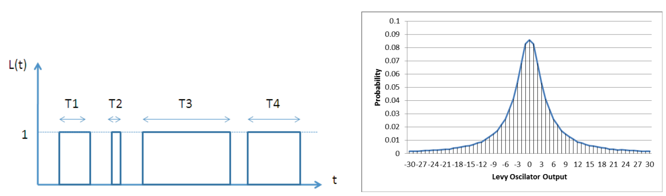


Fig. 2. State diagram of the controller. Here, T_L is the time delay generated from the Lévy distribution generator, and T_U is the time delay generated from the Uniform distribution generator.

converges to the Lévy probability distribution with larger n (the usual value of n is 100 [12]).



(a) (b)

Fig. 3. Output of the Lévy generator **(a)** Oscillator output. **(b)** Probability distribution.

The output of the approximated random number generator is a Pulse Width Modulation (PWM) oscillator that has duty cycle proportional to the Lévy distribution. This PWM output is applied to control the activation of the AUVs propellers, providing forward motion. The propellers are activated during the positive phase for a length of time based on the Lévy random number generator. During the negative (zero) phase the AUV changes its heading (0-360 degrees) based on a uniform distribution.

B. Swim-length generator

The time duration of forward propeller's activation is changed in every random walk phase according to the Lévy distribution. However, as described below, the motion equation of an AUV platform is non linear, as such actual swim length attained has a different probability distribution than the generated oscillator.

The motion equation of the 'Lily' AUV is derived from the drag force equation:

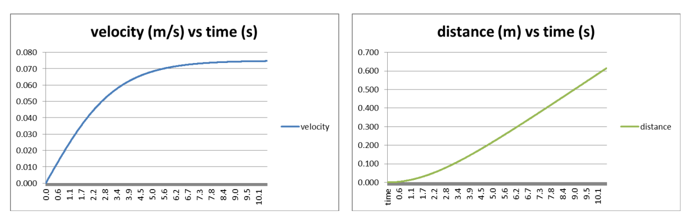
$$m.a = \frac{1}{2}\rho.v^2.A.C_d \tag{7}$$

where m is the mass of the AUV, a is the acceleration, ρ is the water density, v is the velocity, A is the surface area of the frontal part of the vehicle, and C_d is the drag coefficient. Empirical data has revealed $\frac{1}{2}\rho.A.C_d$ to be 0.562, and m to be 440g [8].

By applying maximum thruster force (measured at $0.01N$ [8]), the motion equation of the AUV is a non-linear first order differential equation:

$$\frac{dv}{dt} + 1.4v^2 = 0.025 \tag{8}$$

The non-linear motion equation of the AUV has an exponential solution. Solving the differential equation by using an online numerical method on the small AUV platform is computationally expensive. A less computationally expensive offline solution, such as look-up table, can be used instead. Another alternative method is by eliminating the non-linear part of the motion behavior of the AUV, which disappears after the robot enters the terminal velocity condition. It is also shown in the figure that the sum of the non-linear part is constant if the thrusting force of the propeller is also constant. Nonetheless, the non-linear error is proportional to the change in the thrusting force. The validity of the alternative method can be proved analytically by using a convolution operation; the proportional error does not change the probability distribution. Furthermore, it has been biologically established that some error in the Lévy flight is acceptable with respect to successful foraging activity [5].



(a) (b)

Fig. 4. Dynamic response of the 'Lily' AUV **(a)** Velocity vs Time. **(b)** Distance vs Time.

Fig. 4 shows that terminal velocity is reached after $10s$, having travelled 50cm. Therefore, by adding $10s$ of offset to the generated random variable, the non-linear part of the motion can be eliminated. However, this solution is not feasible for experiments in small swimming pool environments or with experimental aquariums of limited size. Thus, the look-up table solution is adopted here instead. Non-linear motion also occurs with respect to vertical movement (e.g. diving), however the combination of reliable data provided from a pressure sensor and a diving control algorithm [8] can eliminate the non-linear component of vertical movement.

C. *Underwater Robotic Simulation Platform*

The experimental simulation is implemented in CoCoRo's 3D underwater swarm robotic simulator (CoCoRoSim), which is implemented in Netlogo 3D. CoCoRoSim simulates small to medium sized underwater environments, and emulates both sensor/actuator functionalities of the 'Lily' platform, and the physical properties (water density and damping, gravity, etc) of the underwater environment.

A large simulated environment is necessary for implementing swarm robotic experiments with many AUVs. As such an environment representing $8 \times 8 \times 1.5$ meters (height, width, depth) is prepared. Static robots that transmit specific messages over a particular range represent targets of interest that must be located by the swarm. In order to simplify observation, targets change color when located by members of the swarm.

V. Multi-Robotic Random Search Experiments

A. *Simulation-based experiments*

We perform simulation-based experiments to compare the performance of several exploration algorithms for varying numbers of robots in the swarm and differing distributions of targets in the environment (sparsely distributed or clustered). In addition to a Lévy flight algorithm and Lévy flight complemented with adaptation provided through the Firefly algorithm outlined above, random walks based on Gaussian and Uniform distributions are investigated. As these are stochastic algorithms data presented here are representative of 100 individual simulation executions.

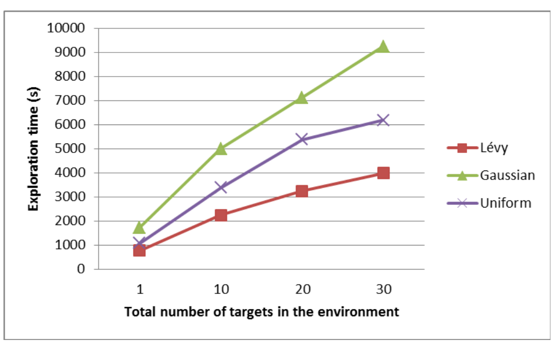


Fig. 5. Locating sparsely distributed targets with a swarm of 20 robots.

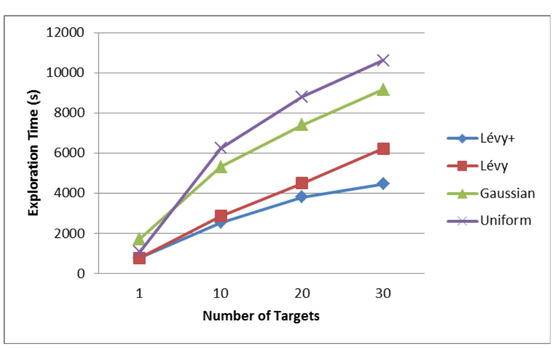


Fig. 6. Locating clustered targets with a swarm of 20 robots.

The first experiment demonstrates the applicability of Lévy flight in locating targets in 3D underwater environments. Here, Lévy flight is compared with Gaussian and Uniform random walks, which employ normal and uniform distributions in selecting the length of walks/swims in each phase.

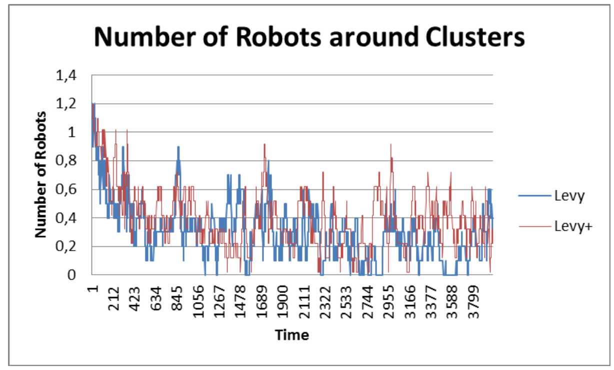


Fig. 7. Number of robots near clusters over time. The average number is 0.29 for the Lévy flight and 0.42 after added with the modified FO algorithm.

Fig. 5 demonstrates that Lévy flight outperforms the other random walk strategies by locating all the targets deployed in an environment in the least time. The other compared algorithms are Gaussian and Uniform random walk. The selected Gaussian random walk has zero mean for exploiting walk-length near-by the robot position and has large variance (which is equal to the size of the environment) for generating longer jump. The Uniform random walk is set to have half-length of the environment for the average walk-length. In this experiment, targets are sparely distributed throughout the environment. Lévy flight's performance is due to the infinite second moment of its distribution, which results in less frequent long jumps to locate targets than Gaussian or Uniform distributions. Fig. 6 demonstrates similar trends when targets are clustered near one another, and further highlights the additional performance increase bestowed upon Lévy flight by introducing the modified FO strategy (see Sec. III).

These results show that the Gaussian distribution has an improved performance in clustered targets than the Uniform distribution, but not in sparsely distributed targets. Lévy flight is more robust to the particular distribution of the targets, offering the best performance in both cases.

Fig. 7 shows time-series data of the number of robots located near clusters for Lévy flight and Lévy flight added with the modified FO (Levy+) exploration strategies. The average number of robots near the clusters increases from 0.29 to 0.42 after applying the fireflies inspired adaptation algorithm. This experiment shows that the modified FO algorithm substantially impacts the adaptability and the collective behavior of all robots, allowing them to remain in the vicinity of targets for longer, and reducing the exploration time for locating all clustered targets.

It is also shown in Fig. 8 that the optimization algorithm significantly decreases the time required to locate all targets, and the benefit is more apparent for greater numbers of targets.

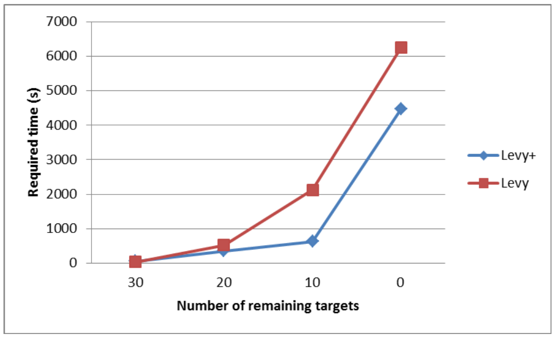


Fig. 8. Targets searching experimental result with 20 robots. Required time vs remaining targets

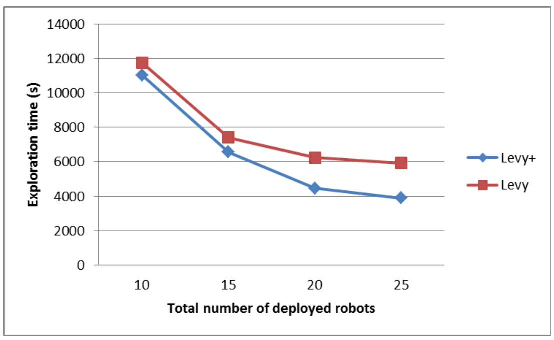


Fig. 9. Targets searching experimental result . Exploration time vs number of robots

In search and rescue scenarios the reduction in average time to locate all targets can be of huge benefit, particularly if lives are at risk. The results of Fig. 9 also highlight that the optimization algorithm has greater impact when larger swarms are deployed. This is likely due to the increases opportunity for communication amongst robots. Without the collaboration that the optimization algorithm provides, increasing the number of robots improves competition among robots and reduces the benefit of additional robots.

B. Real robot experiment

This section describes the deployment of several exploration strategies in real-world robots, in a medium sized aquarium of dimensions 2.5×2.5×1.5× meters.

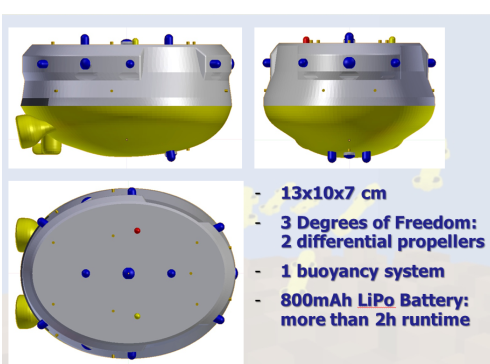


Fig. 10. 'Lily' underwater swarm platform: dimensions and locomotion.

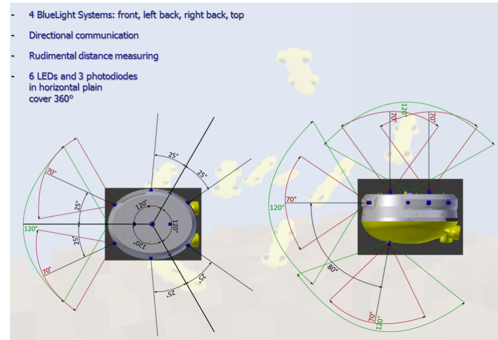


Fig. 11. 'Lily' underwater swarm platform: Sensing angle.

The proposed algorithm is implemented on the 'Lily' CoCoRo underwater swarm platform. 'Lily' is a small AUV of 13x10x7 cm dimensions. Horizontal motion is provided by two differential propellers on the lower left and right (see Fig. 10), and a buoyancy pump provides vertical motion.

'Lily' AUV uses blue-light based optical systems for both sensing and communication. The blue light optical sensor can perform passive sensing for measuring relative distance with other robots, and also to accomplish active sensing for detecting some obstacles in underwater environment. In the 'Lily' AUV, the maximum range is 70cm for passive sensing and communication, and 15cm for active sensing. The maximum bitrate of the optical communication system is 100kbps. The technical specification of the sensing and communication fulfill the requirement of the FO algorithm implementation, that specify longer range for communication than for sensing (attracting range $>$ target sensing range).

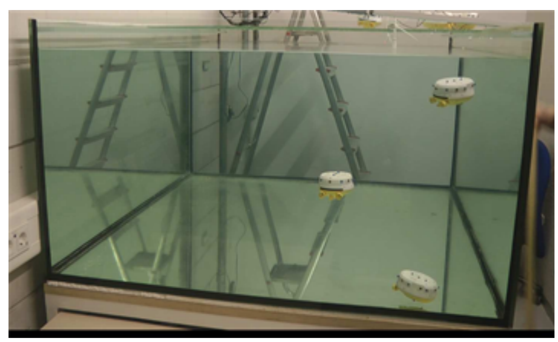


Fig. 12. Real robot experiment with 'Lily' Cocoro platform (rear view)

During the real robot experiments, five robots are deployed into the aquarium to find three passive robots which are sparsely distributed. 10 trials of experiments are done for Uniform, Gaussian, and Lévy flight random walk strategies. Given the limited number of Lily robots available, a clustered targets experiment can not be performed. A simple implementation of the FO algorithm for robot aggregation behavior is implemented separately and tested within the real robot, in order to verify the feasibility of implementing the adaptation capability by using FO algorithm.

Table I demonstrates that, as with the simulation-based results, the Lévy flight strategy out-performs Gaussian and

Type of random walk	Average exploration time
Uniform	357.3 s
Gaussian	393.9 s
Lévy flight +	271.3 s

TABLE I
RESULT FROM REAL ROBOT EXPERIMENT.

Uniform distribution random walk strategies in locating the targets. In this case targets were located in 25% less time than the Uniform walk, and 31% less time than the Gaussian walk.

VI. CONCLUSION AND FUTURE WORK

In our experiments the proposed adaptive random searching algorithm is implemented and investigated for collective underwater exploration scenario. The proposed algorithm is the Lévy flight algorithm that is scaled adaptively by using a modified firefly optimization algorithm.

The result of the experiments with the 3D underwater multi robotic simulator and with the 'Lily' AUV robot show that the performance of the proposed bio-inspired algorithm outperforms Gaussian- and Uniform-distribution based random walk strategies. Simulation experiments also confirm that the added adaptation algorithm improves the performance substantially when targets are located in clusters in the environment. Such a clustering is typical of targets in the natural world, such as prey, human victims following a disaster, and sea mines. The added adaptation method also accelerates the location of targets during the earlier stages of search, and further improves performance when additional robots are deployed in the environment.

From practical point of view, the experiment with the 'Lily' AUV demonstrates the feasibility of implementing the Lévy flight and firefly algorithm to a simple and small underwater robotic platform.

The investigated algorithm shows promise, and is ready for deployment in a real-world underwater exploration problem where knowledge of the environment is limited. For future work, the combination of the optimized bio-inspired algorithm and a 3D simultaneous localization and mapping (3D SLAM) method represents a very promising idea.

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